Analysis of Curved and Angled Surfaces on a Cartesian Mesh Using a Novel Finite-Difference Time-Domain Algorithm

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Abstract— The widely accepted finite-difference time-domain algorithm, based on a Cartesian mesh, is unable to rigorously model the curved surfaces which arise in many engineering applications, while more rigorous solution algorithms are inevitably considerably more computationally intensive. A nonintensive, but still rigorous, alternative to this approach has been to incorporate *a priori* knowledge of the behavior of the fields (their asymptotic static field solutions) into the FDTD algorithm. Unfortunately, until now, this method has often resulted in instability. In this contribution an algorithm (denoted 'SFDTD' for second-order finite difference time domain) is presented which uses the static field solution technique to accurately characterize curved and angled metallic boundaries. A hitherto unpublished stability theory for this algorithm, relying on principles of energy conservation, is described and it is found that for the first time *a priori* knowledge of the field distribution can be incorporated into the algorithm with no possibility of instability. The accuracy of the SFDTD algorithm is compared to that of the standard FDTD method by means of two test structures for which analytic results are available.

I. INTRODUCTION

THE finite-difference time-domain (FDTD) technique is widely accepted as an efficient, reliable, and flexible method for the electromagnetic analysis of a wide variety of structures. Perhaps the most fundamental limitation of FDTD is that in its usual form, first suggested by Yee in 1966 [1], the method represents the modeled object as a Cartesian-based mesh of field components. This spatial discretization prevents the standard FDTD method from accurately characterizing the curved structures which arise frequently in engineering applications.

In [2], the authors first presented a finite-difference time-domain algorithm (second-order finite difference time domain or 'SFDTD') which facilitated the treatment of curved metallic structures. The algorithm utilized the static field solution technique, originally described in [3], to rigorously model the curved surfaces. Employing static field solutions when attempting to analyze curved bodies with the well-known Yee algorithm (FDTD) often resulted in instability; SFDTD, however, appeared not to suffer from this problem.

For clarity, we initially review some of the background pertaining to the modeling of curved structures and then de-

scribe a modification of the SFDTD correction factor scheme given in [2]. This modification results in it being possible to show, by means of a previously unpublished stability theory, that the stability of the corrected algorithm is assured. Further validation of the SFDTD algorithm is then given for the case of angled and curved metal structures.

II. MODELING CURVED STRUCTURES WITH FINITE-METHODS

The conventional approach to modeling curved surfaces with FDTD is to employ a finely staircased mesh [4], this approach is unattractive as it requires a large number of FDTD unit cells and a correspondingly small time-step. An alternative approach is to locally deform the integration contours of the FDTD algorithm [5] in the vicinity of the curved surface; this method yields improved accuracy but may require nonphysical nearest neighbor "borrowing" of field components. For planar circuits, the locally conforming method of Gwarek [6] may be employed.

There are rigorous approaches to the time-domain characterization of curved bodies, those recently proposed include finite-volume [7], hybrid finite-volume/finite-difference [8] and vector finite element methods [9]. These techniques yield much improved accuracy at the expense of increased numbers of operations at each time-step and extra memory requirements.

A different approach has been followed at the Centre for Communications Research, University of Bristol, whereby the normal FDTD method is utilized with correction factors, based on the static field solutions, introduced into the standard difference equations in the vicinity of the curved surface. These factors are calculated by assuming the variation of the field close to a metal object to be dominated by its asymptotic static behavior [10].

This approach can be briefly summarized as follows: A section of the standard Yee mesh, describing the spatial discretization of the electric and magnetic fields, is shown in Fig. 1.

If a metallic boundary intersects the surface of integration of a field component as shown, the standard difference equations for the affected component are modified by the inclusion of altered coefficients (or "correction factors") which are calculated from the field's static behavior.

If the standard FDTD method is viewed as a moment method with delta test functions and piecewise linear basis

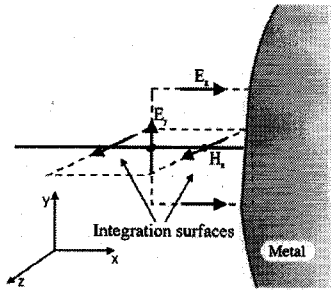Fig. 1. FDTD correction scheme.

Fig. 2. SFDTD correction scheme.

functions, the static field solution theory can be interpreted as the local modification of the linear basis functions to a form which more closely resembles the expected spatial behavior of the fields [10]. This is a standard technique in finite-element analysis where higher accuracy and reduced numbers of basis functions can be achieved in this manner.

Static field solutions have been very successful in permitting the accurate analysis of a number of curved [10] and small-scale features [11]. In addition to its accuracy, the technique requires no increase in computational effort over the standard model, apart from a short initialization procedure within which the correction factors are calculated.

The drawback to this potentially invaluable technique is that instability may result from the introduction of the correction factors into the FDTD algorithm. The problem of whether or not an arbitrary set of correction factors will result in instability does not appear to be amenable to either an analytic or a practical numerical solution and the problem of how that set should be modified to avoid the instability is even more intractable.

III. A NEW FINITE-DIFFERENCE ALGORITHM

In an attempt to solve the instability problem, a model based on the electric field vector wave equation was employed, as first described in [2]; in this algorithm the magnetic fields are eliminated. In addition, it becomes possible to employ a colocated field discretization (i.e., one where all the field components in a given unit cell are placed at the cell's vertices) which leads to a much more elegant correction factor formulation.

Eliminating the magnetic field $\mathbf{H}$ from Maxwell's curl equations in a lossless medium gives

$$\nabla \times (\nabla \times \mathbf{E}) = -c^{-2}\partial_{tt}\mathbf{E} \quad \nabla^2\mathbf{E} - \nabla(\nabla \cdot \mathbf{E}) = c^{-2}\partial_{tt}\mathbf{E} \quad (1)$$

where $\mathbf{E}$ is the electric field, c is the velocity of propagation and t is time. In *uniform media* ($\nabla \cdot \mathbf{E} = 0$) this simplifies to

$$\nabla^2\mathbf{E} = c^{-2}\partial_{tt}\mathbf{E}. \quad (2)$$

(In the case of metallic boundaries the field divergence is nonzero but, in effect, the correction factor scheme described in the next section re-introduces the electric field divergence term).

Using an electric field discretization scheme where all field components are colocated and assuming a regular spatial mesh, the second-order partial derivatives may be replaced by
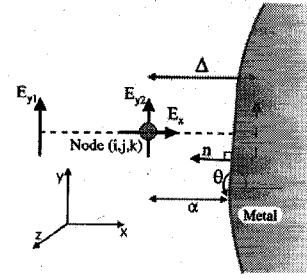
centered difference approximations. For example, the update equation for a field component E_y may be written

$$E_y^{n+1}(i,j,k) = -E_y^{n-1}(i,j,k) + (2 - \frac{6c^2\Delta_t^2}{\Delta^2})E_y^n(i,j,k)$$
$$+ [E_y^n(i+1,j,k) + E_y^n(i-1,j,k)$$
$$+ E_y^n(i,j,k+1) + E_y^n(i,j,k-1)$$
$$+ E_y^n(i,j+1,k) + E_y^n(i,j-1,k)]\frac{c^2\Delta_t^2}{\Delta^2} \quad (3)$$

where Δ is the space-step, Δ_t is the time-step, and $^n(i,j,k)$ represents a point in space $(i\Delta, j\Delta, k\Delta)$ at time $t = n\Delta_t$.

Update equations for the other electric field components may be derived similarly, yielding a second-order finite difference time domain algorithm (or 'SFDTD' for convenience) as opposed to the standard FDTD algorithm which involves only first-order derivatives. This discretization of the wave equation is well known, being nothing more than, for example, the extension to three dimensions of the one-dimensional algorithm of [12]. Other, similar, algorithms are those of [13] and [14]. The new aspects of this contribution are the modifications, described in Section IV, which enable the rigorous treatment of curved surfaces and the stability theory developed in Section V.

It can be shown that the Courant stability criterion (which relates the maximum time step Δ_t to the minimum space step Δ) is identical to that required for FDTD. The extra amount of memory required by SFDTD to store the past value of each electric field component (E_y^{n-1} in (3)) is balanced by that needed for the storage of the magnetic fields in FDTD. The computational effort associated with SFDTD in terms of the number of numerical operations is slightly lower than that of FDTD.

IV. SFDTD CORRECTION FACTOR TECHNIQUE

A curved metal surface may be accurately approximated on a small scale by an angled planar surface as shown in Fig. 2. The behavior of the electric field close to a metal boundary is well known to converge to the static solution [15] and hence, in this case, may be described by two functions

$$E_n = k_1 \quad E_t = k_2n \quad (4)$$

where n and t are coordinates normal and tangential, respectively, to the surface.

Taking as an example the first-order spatial derivative

$$\partial_x E_y = \left[\partial_n E_y \frac{\partial n}{\partial x} + \partial_t E_y \frac{\partial t}{\partial x} \right] \tag{5}$$

since

$$n = x \sin\theta - y \cos\theta$$
$$t = -x \cos\theta - y \sin\theta \tag{6}$$

and

$$E_y = -E_n \cos\theta - E_t \sin\theta \tag{7}$$

we produce an expression within which (4) may be substituted, yielding the following improved expression for the derivative at the metal boundary

$$\partial_x E_y \big|_{\text{improved}} = -(\sin^2\theta)k_2. \tag{8}$$

However

$$E_t = -E_{y2} \sin\theta - E_x \cos\theta = k_2 n_0 \tag{9}$$

where n_0 is the normal distance from the position of E_{y2} and E_x to the metal boundary (see Fig. 2). Thus

$$\partial_x E_y \big|_{\text{improved}} = \frac{(\sin^2\theta)(E_{y2} \sin\theta + E_x \cos\theta)}{n_0} \tag{10}$$

the approximation used to the second-order derivative is

$$\partial_{xx} E_y(i,j,k) = \frac{\partial_x E_y(i+1/2,j,k) - \partial_x E_y(i-1/2,j,k)}{\Delta} \tag{11}$$

(11) represents a modification of the expression (9) in [2], where the denominator used was $\frac{1}{2}(\Delta + \alpha)$ in an attempt to centre the derivatives at the point (i,j,k). In fact, doing this makes little difference to the accuracy of the results and prevents the use of the stability theory given in the following section.

Replacing $\partial_x E_y(i+1/2,j,k)$ with (10), and $\partial_x E_y(i-1/2,j,k)$ with the usual centered difference approximation, the required expression for the second-order derivative is

$$\partial_{xx} E_y(i,j,k) =$$
$$\left(\frac{E_{y1}}{\Delta^2} - \frac{E_x \cos\theta \sin\theta}{\Delta^2 \beta} \right) - \frac{E_{y2}}{\Delta^2} \left(1 + \frac{(\sin\theta)^2}{\beta} \right) \tag{12}$$

where β is defined as α/Δ and E_{y1}, E_{y2} and E_x are field components neighboring the boundary (see Fig. 2).

Again, (12) differs slightly from that given in [2] for the reasons described above.

This corrected discrete approximation may be utilized instead of the standard difference form in the SFDTD algorithm. If β is unity and $\theta = 270°$ then the original difference form is returned and, unless $\sin\theta$ is 0 or 1, energy will couple between orthogonal field components (as expected).

The approximations which have been made are that the boundary may be approximated over the unit cell by a planar surface (a considerable improvement over its approximation by a staircase) and that the fields will assume their static forms over a distance $\leq \Delta$ from the boundary (which is reasonable if Δ is a small fraction of a wavelength).

V. STABILITY THEORY FOR THE NEW ALGORITHM

In [2] it was stated that the SFDTD algorithm with its curved surface correction factors was an inherently more stable algorithm than the corrected FDTD algorithm. At the time of publication, however, the reasons for this were not well understood.

A necessary criterion for the stability of a numerical model is that it is a model of a stable physical process, this is intuitively clear and need not be discussed further except to say that this criterion is not by itself sufficient, as shown by the well-known limit on the FDTD time-step Δ_t [16] which, when violated, results in algorithmic instability despite the energy conservation implicit in Maxwell's equations. We now show that the aforementioned criterion is met by the corrected SFDTD algorithm in two spatial dimensions, the extension to three dimensions being trivial.

In finite-difference form, the uncorrected 2-D SFDTD algorithm for any given field component can be written

$$E^{n+1}(i,k) = -E^{n-1}(i,k)$$
$$+ (2 - k(1+1+1+1))E^n(i,k) + kE^n(i-1,k)$$
$$+ kE^n(i+1,k) + kE^n(i,k-1) + kE^n(i,k+1) \tag{13}$$

with the stability factor $k = \left(\frac{c\Delta_t}{\Delta}\right)^2$ (≤ 0.5 for stability). In (13), as elsewhere in this section, some terms have not been collected together in order to help make clear the correspondences between terms in the update equations and features of the physical problem.

In general, two curved surface corrections may be required as shown, for example, at node (0,0) in Fig. 3, with the angles of the two tangents to the curved boundary being θ_1 and θ_2 and the distances from node (0,0) to the boundary being $\alpha_1 = \beta_1 \Delta$ and $\alpha_2 = \beta_2 \Delta$ in the x and z directions, respectively. The update equations for E_x and E_z at the point $(0,0)$ then become

$$E_x^{n+1}(0,0) = -E_x^{n-1}(0,0)$$
$$+ \left(2 - k \left(\frac{\sin^2\theta_1}{\beta_1} + \frac{\sin^2\theta_2}{\beta_2} + 1 + 1 \right) \right) E_x^n(0,0)$$
$$+ kE_x^n(1,0) + kE_x^n(0,1) - k\frac{\sin\theta_1 \cos\theta_1}{\beta_1} E_z^n(0,0)$$
$$- k\frac{\sin\theta_2 \cos\theta_2}{\beta_2} E_z^n(0,0)) \tag{14}$$

and

$$E_z^{n+1}(0,0) = -E_z^{n-1}(0,0)$$
$$+ \left(2 - k \left(\frac{\cos^2\theta_1}{\beta_1} + \frac{\cos^2\theta_2}{\beta_2} + 1 + 1 \right) \right) E_z^n(0,0)$$
$$+ kE_z^n(1,0) + kE_z^n(0,1)$$
$$- k\frac{\sin\theta_1 \cos\theta_1}{\beta_1} E_x^n(0,0) - k\frac{\sin\theta_2 \cos\theta_2}{\beta_2} E_x^n(0,0). \tag{15}$$

Now consider the passive network shown by Fig. 4; this circuit consists of two separate 2-D networks, one with nodal voltages represented by $V_x(i,k)$ and the other, $V_z(i,k)$, where the coordinates (i,k) specify the voltage at the ith node in the direction x in the network, and the kth node in direction z.
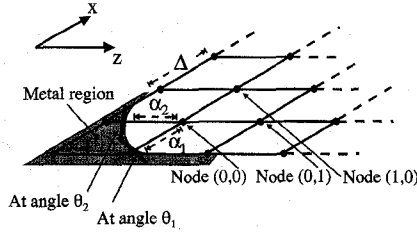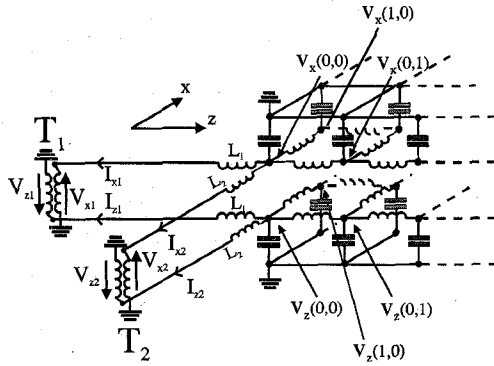
Fig. 3. Example 2-D SFDTD correction scheme.

Fig. 4. 2-D electrical network.

The only connections coupling the two networks are the ideal transformers T_1 and T_2 at the node $(0,0)$. All capacitors are assumed to be identical, with value C, as are all the inductors, L, with the exception of the components L_1 and L_2.

At, for example, the node $(1,1)$ (not connected to the transformer) simple analysis shows that

$$
\partial_{tt} V(1,1)
= \left(\frac{V(2,1) - V(1,1)}{CL} + \frac{V(1,2) - V(1,1)}{CL} \right.
$$
$$
\left. + \frac{V(0,1) - V(1,1)}{CL} + \frac{V(1,0) - V(1,1)}{CL} \right). \quad (16)
$$

Replacing the *temporal* derivative of V with the appropriate centered difference expression yields

$$
V^{n+1}(1,1)
= -V^{n-1} + \left(2 - \frac{\Delta_t^2}{C} \left(\frac{1}{L} + \frac{1}{L} + \frac{1}{L} + \frac{1}{L} \right) \right) V^n(1,1)
$$
$$
+ \frac{\Delta_t^2}{CL} (V^n(2,1) + V^n(1,2) + V^n(0,1) + V^n(1,0)). \quad (17)
$$

Thus, the nodal equations in the region *not* connected to the transformer are identical in form to the SFDTD update (13) in free space with the following substitutions

$$
C = \epsilon \Delta^2 \quad (18)
$$
$$
L = \mu.
$$

It should be noted that the components in the network are analogs of the assumed spatial dependence of the fields in the SFDTD algorithm (i.e., piecewise linear) and that, in the region not connected to the transformer, this two dimensional lumped

equivalent circuit is the same as the planar FDTD equivalent circuit presented in [6].

The analysis of the nodal voltages at the node $(0,0)$, where two of the branch connections are to transformers rather than adjacent nodes, is more involved. Firstly we assume the transformers are ideal and have winding ratios $1 : N_1$ and $1 : N_2$, respectively. Thus for T_1

$$
V_{z1} = -\frac{V_{x1}}{N_1} \quad (19)
$$
$$
I_{z1} = N_1 I_{x1}
$$

since the transformer is ideal. Summing voltages around the loops containing the windings gives

$$
V_x(0,0) - V_{x1} = L_1 \partial_t I_{x1} \quad (20)
$$
$$
V_z(0,0) - V_{z1} = L_1 \partial_t I_{z1}
$$

solving for the nodal voltages and current I_{x1} yields

$$
\partial_t I_{x1} = \frac{V_x(0,0) + N_1 V_z(0,0)}{L_1(1 + N_1^2)} \quad (21)
$$

thus

$$
\partial_t I_{z1} = N_1 \frac{V_x(0,0) + N_1 V_z(0,0)}{L_1(1 + N_1^2)}. \quad (22)
$$

If we now let $N_1 = \cot \theta_1$

$$
\partial_t I_{x1} = \frac{\sin^2 \theta_1 V_x(0,0) - \sin \theta_1 \cos \theta_1 V_z(0,0)}{L_1}
$$
$$
\partial_t I_{z1} = \frac{-\cos^2 \theta_1 V_z(0,0) + \sin \theta_1 \cos \theta_1 V_x(0,0)}{L_1}. \quad (23)
$$

Identical expressions (but with θ_2 and L_2) can be produced for the other transformer. These currents can be used to derive

$$
V_x^{n+1}(0,0) = -V_x^{n-1}(0,0)
$$
$$
+ \left(2 - \frac{\Delta_t^2}{C} \left(\frac{\sin^2 \theta_1}{L_1} + \frac{\sin^2 \theta_2}{L_2} + \frac{1}{L} + \frac{1}{L} \right) \right) V_x^n(0,0) +
$$
$$
\frac{\Delta_t^2}{CL} V_x(1,0) + \frac{\Delta_t^2}{CL} V_x(0,1) - \frac{\Delta_t^2 \sin \theta_1 \cos \theta_1}{CL_1} V_z^n(0,0)
$$
$$
- \frac{\Delta_t^2 \sin \theta_2 \cos \theta_2}{CL_2} V_z^n(0,0) \quad (24)
$$

and it is now clear that substitution of

$$
L_1 = \mu \beta_1
$$
$$
L_2 = \mu \beta_2 \quad (25)
$$

yields the SFDTD update-equation (14) with the curved surface corrections.

It has now been shown that the SFDTD model both with and without the curved surface correction factors is exactly equivalent to a representation of a passive network. Energy conservation is guaranteed in such a network, and so therefore is stability. It is if course possible to have a stable *active* network but in such a case stability can only be assured by examination of all possible feedback paths within the network, this corresponds to the impractical task of evaluating all the eigenvalues of the difference algorithm.
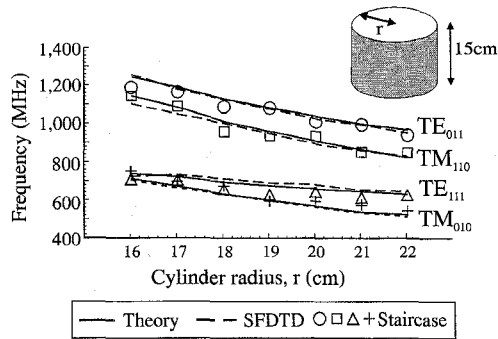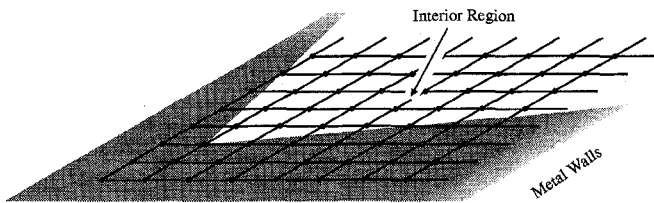
Fig. 5. Cylinder resonant frequencies.

TABLE I
RESONANT FREQUENCIES FOR ROTATED RECTANGULAR BOX

Mode	Theory	FDTD	SFDTD	FDTD Error	SFDTD Error
101	633 MHz	620 MHz	622 MHz	2.1%	1.7%
012	869 MHz	648 MHz	870 MHz	2.4%	0.1%
011	549 MHz	575 MHz	545 MHz	4.7%	0.7%

TABLE II
RESONANT FREQUENCIES FOR A NUMBER OF ROTATED RECTANGULAR BOXES

Angle, ϕ	Mode	FDTD Error	SFDTD Error
11.3°	101	1.6%	0.6%
	012	0.1%	0.6%
	011	6.3%	0.9%
14.0°	101	2.1%	1.7%
	012	2.4%	0.1%
	011	4.7%	0.7%
18.4°	101	0.3%	1.8%
	012	0.6%	0.8%
	011	11.3%	0.5%
26.6°	101	3.7%	2.1%
	012	4.2%	1.1%
	011	8.2%	1.0%

Fig. 6. Segment of rotated rectangular box cross-section for $\phi = 14°$.

Here it should be noted that if the denominator of (11) is that given by (9) in [2] it is not possible to produce a passive circuit equivalent to the corrected algorithm. This implies that the slightly different algorithm given in [2] may exhibit instability, and for a few structures this has proved to be the case.

VI. VALIDATION OF THE NEW ALGORITHM

A. Test Case 1—Cylindrical Resonator

A metal-walled closed cylindrical resonator identical to that described in [10], was modeled using the combination of SFDTD and the correction method described above and a *fixed* uniform mesh size of 5cm. The simple geometry allows analytic cavity resonance techniques to predict the resonant frequencies with high accuracy for comparison.

Fig. 5 shows the variation in the cylinder's resonant frequencies as a function of radius. The solid line represents the (perfect) analytic solution and the dashed line the results produced by the new algorithm, SFDTD. The marker points indicate the predictions of the standard, staircased, FDTD method (employing the same mesh size). Comparison of these results with those given in [2] using the slightly different, and potentially unstable, scheme shows that the modification to the correction factors introduced in Section IV of this contribution has had little effect on the results. Indeed, particularly for the model with radius 16 cm, some improvement in accuracy can be noted.

Overall, given the coarseness of the mesh with respect to both frequency and surface curvature, the SFDTD results adhere well to the theoretical curves and are, as expected, considerably more accurate than the staircased FDTD technique. The TM_{010} mode in particular is excellently characterized, virtually independent of cylinder radius. The results for the TM_{110} mode are good for radii of >18 cm but become less accurate as the radius decreases (this is probably due to both

a decrease in the number of field components available to describe the cylinder and to the increase in frequency of the mode). The most difficult mode to model is clearly the TE_{111} mode, the FDTD results for this are notably poor and while the SFDTD algorithm performs more consistently, the results may indicate potential for improvement in the technique.

B. Test Case 2—Rotated Rectangular Box

A metal walled rectangular box with square cross section was analyzed, again with a fixed mesh size of 5 cm and a height of 15 cm, however, the box was rotated through an angle ϕ with respect to the mesh. This resulted in the sides of the box not being aligned with the nodal-planes of the difference algorithm. For convenience, values of ϕ producing integer gradients were chosen and the box side lengths were selected for each ϕ such that the surfaces of the box passed through the corners of the unit cells - as illustrated for $\phi = \arctan \frac{1}{4} = 14.0°$ by Fig. 6.

The FDTD algorithm approximates the angled surfaces with a staircase and the SFDTD algorithm with the correction factors described above.

Table I shows the resonant frequencies for the box with angle $\phi = 14.0°$ (it should be noted that due to the square cross-section of the structure each resonance may represent more than one mode).

A summary of the results for four angles ($\arctan \frac{1}{5}$, $\arctan \frac{1}{4}$, $\arctan \frac{1}{3}$ and $\arctan \frac{1}{2}$) is given by Table II.

Once again the SFDTD results (with curved surface corrections) agree well with the analytic results, in the majority of cases the resonant frequencies are correct to within one percent. The FDTD algorithm, as might be expected, fares less well, although some modes are well characterized, most exhibit significant error.

The mean error across all modes and all ϕ for FDTD was 3.8% and 1.0% for SFDTD, thus SFDTD's curved surface corrections reduce the modeling error in this case by around a factor of 4; this being a similar figure to that achieved

when modeling the cylindrical cavity described in the previous section.

As expected, neither the cylindrical nor the rotated-box geometries exhibited any form of numerical instability.

VII. SUMMARY AND CONCLUSION

This contribution has shown how an alternative finite-difference time-domain algorithm can be derived. This algorithm, SFDTD, can be simply modified to rigorously model both curved and angled metal surfaces. The algorithm's stability is assured as the behavior of the field components in the algorithm is an exact analog of the voltages in a passive electrical network.

The SFDTD algorithm is, in the authors' opinions, far better suited to the analysis of curved and angled boundaries than the FDTD method. This fact arises because the colocation of the field components enables a simple resolution into the normal and tangential components in terms of which the boundary conditions are specified.

Future development of this algorithm is expected to include its application to problems containing dielectric interfaces and sharp metallic boundaries (for example microstrip). For each of these cases a specific correction must be introduced into the algorithm by means of the appropriate static field solution, in order to compensate for the loss of the field divergence term, just as has been done here for smooth conducting boundaries. If this can be achieved SFDTD may prove in the future a superior alternative to the established FDTD algorithm.
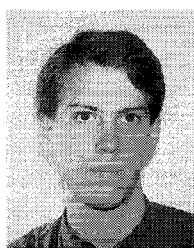
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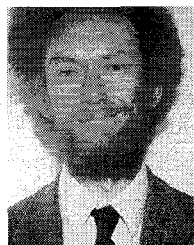
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